# Texture Mapping using Multiperiodic Function on the Smooth Genus *N* Object

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## ABSTRACT

This paper presents a new way of texture mapping on the Genus N object constructed over a single domain. The problem of 2D texture mapping is the discontinuity of texture domain at the virtual boundary on the object. Such phenomenon decreases smoothness of the object as well as looks unnatural. Especially it is necessary for the Genus N object of infinite coninuity to apply the seamless texture mapping. For seamless texture mapping, a multiperiodic function, which transforms a discontinuous function into a continuous function, is suggested. In some applications, however, the visual seams on the textured object provide more realistic appearance. Therefore, this research supports the interactive control from the discontinuity and the continuity across the boundary using the coefficient of the weight function.


# Multiperiodic 함수를 이용한 Smooth Genus *N* 객체의 텍스쳐매핑

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## 요 약

본 연구는 하나의 도메인을 기반으로 하여 모델링된 Genus N 객체에 새로운 텍스쳐 매핑방법을 제안한 연구이다. 기존의 2D 텍스쳐 매핑의 문제점은 텍스쳐 도메인의 경계선이 객체상에 현저하게 나타난다는 사실이다. 이 현상은 부자연스러울 뿐 아니라 객체의 부드러운 연속성의 효과를 감소시킨다. 특히 무한대의 연속성을 가진 Genus N 객체는 경계선이 생성되지 않는 텍스쳐 매핑이 필수적이다. 이러한 텍스쳐 매핑을 위하여 multiperiodic 함수 즉, 불연속의 함수를 연속함수로 변형시키는 함수를 제안하였다. 하지만 사례에 따라 경계선이 보이는 텍스쳐가 더 사실적으로 보일 수 있다. 따라서 본 연구에서는 가중치를 이용하여 불연속과 연속 함수를 상호적으로 제어하도록 하였다.


**Key words:** seamless texture mapping, genus N object, multiperiodic function, hyperbolic plane

## 1. Introduction

Texture mapping is a method of enhancing the richness of computer-generated images. Many texture techniques including marble texture, the bark on a tree, cloud, surface perturbation, and hair of a bear have been developed and classified in several ways based on the texture domain such as 1D texturing, 2D texturing, 3D texturing, environment mapping, and so on. Bump mapping, which is categorized as 2D texturing, is one of the important texture techniques because it simulates a wrinkled or dimpled surface. Generally, the difficulties that occur in texture mapping arise from the non-linear transformation between texture domain and object space. 1D and 3D texture domains have the advantage that the mapping is trivial, but the

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textual effects that can be obtained are generally much more restricted than in two-dimensional texture domains where any two-dimensional image can be used as a texture pattern.

Much research for texture mapping on three-dimensional surfaces has been developed and most of them have worked on patch-oriented objects. In aspects of designing and modeling 3D objects, it is true that such patch-oriented objects have many applications in industry fields. But they don't fit all applications. Especially much complicated computation is required for building smooth genus N objects. In addition, patch-oriented objects occurs many problems in texturing on the surface, which are distortion and discontinuity at the boundary of the patches. Therefore, topological design approach based on the single patch is a way of reducing the complicated computation for smooth sculptures of N genus. Multiperiodic functions are developed practically to make the surface smooth at the virtual cut, boundary. More details are described in [15]. For providing qualitative sculptures, the genus N object raises the same problem, which is the discontinuity at the virtual cut. In this matter, this paper focuses on seamless texture mapping to the genus N object that is constructed on the single patch using multiperiodic functions.

Recently much research has been processed on seamless texturing onto the surface. Brief several texture techniques are reviewed. They can be categorized into texture placement, procedural texture synthesis, and texture synthesis from samples. The methods are aiming at the seamless texture with low distortion and stretching of the pattern. For the texture placement, Maillot et.al [7] presented a method of reducing such distortion and discontinuity between two adjacent surfaces, by considering a general length based energy minimization. These problems are basically coming from the patch oriented objects, i.e. object consisted of patches. In addition, Levy and Mallet[12], Pedersen[13] can be included in this field. For procedural texture synthesis, wood grain, marble, or water wave textures on the surface are created by procedural means using 3D noise functions. Reaction-diffusion is also a way of generating patterns of spots and stripes that can be adopted on animal polygons. Sometimes it generates wonderful image but has the limitation that requires a programmer to test code until the result is satisfied. The approach that is developed recently is the texture synthesis from samples. It allows the user to supply a small patch of texture and creates more texture that looks similar to this sample. In order to generate the new texture, Heeger and Bergen[9] make use of Laplacian and steerable pyramid analysis of a texture sample, and De Bonet [11] utilized multi-scale pyramid analysis to perform synthesis. Efros and Leung [10] suggest the use of best matches for the pattern of surrounding pixels of the visited pixel. Wei and Levoy [14] improved Efros and Leung's method by employing multi-scale pyramids and speeded it up by using vector quantization. In most cases, they generate wonderful results on a regular grid of pixels. But they have the limitations, which are the fact that they just handle patterns instead of images and the fact that the texture is generated on the rectangular domain.

The proposed method in this paper can be classified into the texture placement in terms of texture domain. Since the color on texture domain is placed on the surface of the 3D object with the same parameters. But, the texture domain is changed according to number of genus. The 8-gon is used as the domain for the genus 2 objects, the 12-gon is for the genus 3 objects, and so on. This research is aiming at the design of the textured sculpture, an example of genus N objects, without any visible cutting line. Just wrapping the texture on the surface makes the virtual cut visible. In order to make it invisible, we generate a multiperiodic texture function. For this reason, the design approach is based on the genus N object built on the single patch.

This paper suggests a new way of texture mapping using multiperiodic function on genus N objects that are also based on the multiperiodic approach over a single patch. In addition, this paper suggests a way of control from the seamless cut to visible cut across the boundary. In some applications, it is desirable to have a visible seam line, e.g. cloth manufacturing (see Bennis et.al [2]), while the clay modeling in a sculpture application area prefers a seamless object rather than a visual cut. Hence, it would be better to provide an interactive control for adjusting from the visual cutting object to the seamless object across the boundary. This paper is organized as follows: In section 2, basic concepts on the topological design and multiperiodic function are described in a brief manner, Section 3 contains a texture function using multiperiodic function on the genus N object. Bump mapping using multiperiodic function on the genus N object is explained in section 4, and finally, conclusion with future works is following in section 5.

## 2. Basic Concepts

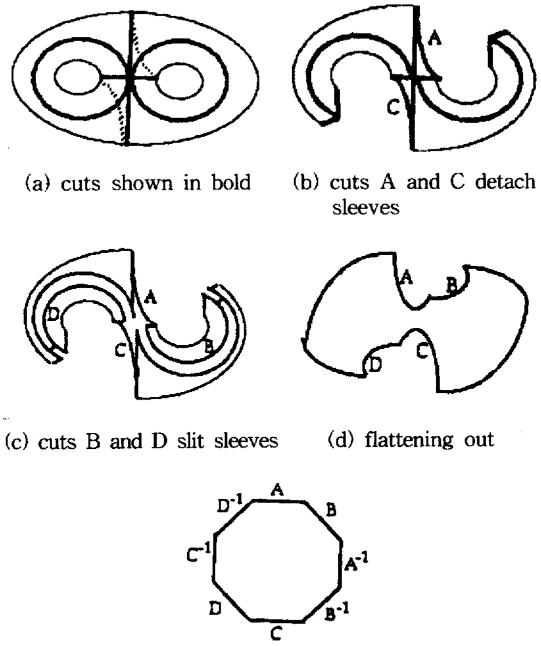
### 2.1 Topological Design for Genus N Objects

In topology, orientable compact surfaces can be classified in terms of the numbers of "handles" on them. Here, the number of handles in the object is defined by its genus. A sphere is classified as genus 0, since there is no handle on it. A torus with one handle is a genus 1 object. In word representation, all genus 0 objects are represented as $aa^{-1}$ and all genus 1 objects are represented as $aba^{-1}b^{-1}$. All genus $n$ objects are represented as $a_1b_1a_1^{-1}b_1^{-1}a_2b_2a_2^{-1}b_2^{-1}...a_nb_na_n^{-1}b_n^{-1}$. This way of representation can be derived from a classification theorem, which is defined as follows [5].

**Classification Theorem** An orientable compact surface of genus $n$ object is classified into $n$T for some $n \geq 0$, where T represents a torus i.e. $aba^{-1}b^{-1}$.

A corollary to the theorem is that the surface of the genus $n$ object can be mapped into a $4n$-sided polygon in 2D, since one cut always becomes two boundaries and each torus has two cuts. Fig. 1 shows the octagon that is mapped from a double torus by cutting and unfolding it into the plane.

Any two objects having the identical genus are called *topologically equivalent*. Certain mappings such as bending, stretching, and squashing without tearing or 'gluing' points together do not affect the genus of it. So, if a surface is deformed to another surface by using bijective, bicontinuous mappings, these surfaces are topologically equivalent and they are called *homeomorphic*. For example, a coffee cup with a handle is homeomorphic to a torus. All objects with the same genus can be called *homeomorphic*. Formal details are described in [5].



(a) cuts shown in bold (b) cuts A and C detach sleeves
(c) cuts B and D slit sleeves (d) flattening out
(e) stretch into polygon form

Fig. 1. Peeling a double torus

### 2.2 A Multiperiodic Function for Genus N Objects

To design a circle using a polynomial $f(t)$ on the one-dimensional domain is the simplest example that can retain all the basic properties of this topological design approach. A circle can be cut and

unfolded, in which case it will become a line segment. The cut on a circle corresponds to a single point. The line segment is normalized to (-0.5, 0.5) on the real line and is considered as the fundamental polygon in 1D. The real line-the whole domain in 1D-can be tessellated by the translation of this fundamental polygon by adding or subtracting 1. This line segment is used as a common parametric domain of a circle. Given the line segment (-0.5, 0.5), the mapping from the line segment to a plane (-0.5, 0.5)×(-0.5, 0.5) may generate a self-intersection curve or an *embedded* curve without a self-intersection. This research is focused on the embedded curve since all natural objects are embeddings, not self-intersections. For instance, two parametric functions $x(t) = 1 - 16t^2$ as the $x$ coordinate function and $y(t) = t(1-4t^2)$ as the $y$ coordinate function are taken. This polynomial function $f(t) = (x(t), y(t))$ has a form of a tear drop curve with a corner on the interval (-0.5, 0.5), which is neither periodic nor smooth. Fig. 2 (a) depicts a teardrop curve. In order to make the curve infinitely smooth at the corner, a periodizing transformation is applied to the polynomial $f(t)$ by convolving with a basis function that is rapidly convergent. Generally, it is written as

$$F(t) = \sum_{k\in\Gamma} f(t+k)w(t+k) \quad (1)$$

where $f(t)$ is a guess function for approximating a desired shape and $w(t)$ denotes a basis function for periodizing $f(t)$ and $\Gamma$ is the set of all the periodic transformations in the domain. The resulting $F(t)$ is the infinite sum of the multiplication of $f(t)$ and $w(t)$ over all the given periods.

In this simple example, since the domain is a real line in 1D and tessellated by the line segment with the length 1, $\Gamma$ is the set of integers Z and each period can be generated only by adding 1. Rewriting the equation (1) with the gaussian function $e^{-t^2}$ as the basis function, it is as follows

$$F(t) = \sum_{k\in Z} f(t+k)\, e^{-(t+k)^2}$$

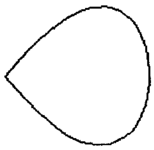
which is periodic with period 1. Therefore, $F(t) = F(t+k)$ for all $k$ in Z. Applying the polynomial functions $x(t)$ and $y(t)$ into $f(t)$ will lead to,

$$F\begin{pmatrix} x \\ y \end{pmatrix} = F\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \sum_{k\in Z}\begin{pmatrix} x(t+k) \\ y(t+k) \end{pmatrix} e^{-(t+k)^2}$$

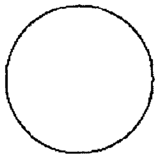
Rescaling $x(t)$ and $y(t)$ into the [0, 1]×[0, 1] and taking only five terms in the sum gives cosine and sine functions to eleven digits; almost a circle, as illustrated in Fig. 2(b). Moreover continuity at cuts is not a requirement for the guess function.

The general procedure for designing a smooth object follows as:

1) Begin with an idea of what to draw,
2) Shape the object by designing the coordinate functions that give an embedding
3) Make the function periodic by convolving it with the converging function over a set of "translations"



(a) A polynomial embedding of a circle



(b) A periodic embedding of a circle

Fig. 2. Periodizing transformation

The same philosophy in designing the circle can be applied to construct the *n* torus which is a genus *n* object, where *n*>1. According to the classification theorem, the genus *n* object is mapped to a 4*n*-sided polygon, for instances, an 8-gon for a genus 2 object and a 12-gon for a genus 3 object. As it is already shown in circle example, periodizing an interpolating function is necessary in order to make the virtual cutting line smooth. Since this research is dealing with 4*n*-sided polygons and the regular tessellation using the 4*n*-sided polygons, the Euclidean plane, where the {4, 4} regular tessellation for a torus works well, is not appropriate for a genus *n* object. Instead, the hyperbolic plane al-

lowing an infinite number of tessellations including {8, 8}, {12, 12}, etc. is used. More details on hyperbolic plane are contained on [6,8].

The general design procedures are as follows.

- Selection of the parameters $(u_i, v_i)$ on the fundamental polygon in the hyperbolic plane and their corresponding data points $(x_i, y_i, z_i)$, $i$=1,...,$n$ which go through a desired object in object space.
- Triangulation the parameters on the fundamental polygon for manipulating the given data points in object space.
- Generation of the scattered data interpolating function over the hyperbolic domain.

Fig. 3 shows the genus 2 object (double torus) that is constructed by above procedure. More details on topological design approach and multiperiodic functions are described in [3,4], while the interactive design with it is suggested in [15].

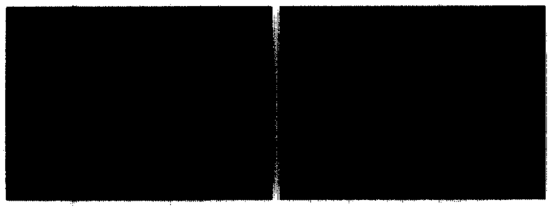


Fig. 3. multiperiodizing transformation

## 3. Multiperiodized texture function on the Genus N Object

This paper is aiming at the seamless texture mapping and bump mapping on the genus N object. If the texture is generated from a function, it is similar to surface on surface. Since the topological design approach defines the genus N object as a global function in form of a multiperiodic function, the texture function on the genus n object can be represented as a function. It is natural for a smooth object with dimps to have no seam on it, for examples, an orange, a strawberry, and, a golf ball, etc. But the visible seam on the object is desirable in some applications such as cloth manufacturing.

A periodizing function transforms a discontinuous function into an infinitely continuous function. In order to make a seamless texture on the genus n objects, which is very difficult in the patch oriented object environment, the texture function on the surface can be periodized. If the periodizing is applied in 1D (i.e. a circle) which domain is a line, it is called "periodizing". If it is applied in 2D (i.e. a torus), which domain is rectangle, it is called "doubly periodizing". Since it is dealt with genus n objects, which domain is 4n gon, it is called "multiperiodizing". This section begins with a periodized texture function on a line and finally describes multiperiodized texture function on the genus N object.

### 3.1 Periodizing of a Texture Function

Consider a circle as the simplest example. The discontinuity on the circle with the texture function is shown clearly in Fig. 4 (a). It is represented as a function $o(t)$ by adding the texture function $T(t)$ to the circle $f(t)$ in the normal direction $f'(t)$ at each point.

$$o(t) = f(t) + T(t)\frac{f'(t)}{|f'(t)|}$$

where $f(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} 1-16t^2 \\ t(1-4t^2) \end{pmatrix}$,

$$T(t) = \begin{cases} c & 2n \le t < 2n+1 \text{ for all integer n} \\ 0 & 2n+1 \le t < 2n+2 \text{ for all integer n} \end{cases}$$

where c is constant.

Fig. 4(a) shows $o(t)$ on the domain (-0.5, 0.5), which is tear drop curve with the texture function. Note here that the points of $o(t)$ on $t$ = -0.5 and $t$ = 0.5 are not connected. This results in the visible seam that is not desirable in mapping the texture into the smooth genus $n$ object.

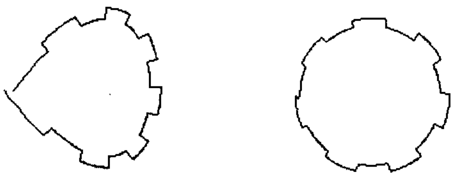
Now, the interactive control from discontinuity to continuity can be developed. This is achieved by transforming a discontinuous texture function

into a continuous texture function as the tear drop curve is transformed to a circle in section 2. As it is described, the continuous texture function $O(t)$ is accomplished by periodizing the discontinuous texture function $o(t)$. The summation of the converging series using a Gaussian function makes any function periodic in a given period. Applying it to the texture function $o(t)$, it is

$$O(t) = \sum_{k\in Z}\left[f(t+k)\right] e^{-a(t+k)^2} + \sum_{k\in Z}\left[T(t+k)\right] e^{-a(t+k)^2} \frac{n(t)}{|n(t)|}$$

where Z is all integers and $\frac{n(t)}{|n(t)|}$ is the unit normal vector at t.

Fig. 4(b) shows a periodized texture function $O(t)$ that is seamless at the boundary. The function value at the boundary, $O(-0.5)$ and $O(0.5)$, has tendency to converge to the average value between $o(-0.5)$ and $o(0.5)$ of the original texture function. Therefore, the texture function is gradually spreading out as it is close to the cutting boundary.

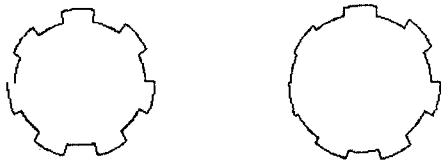


(a) A teardrop curve with a texture function (b) A periodized function from (a)

Fig. 4. A discontinuous texture function and its periodic function

## 3.2 The Effects on the Coefficient in a Gaussian Function

How fast the boundary of two sides is converged to each other is dependent upon the constant $a$ in the Gaussian function. In other words, when $a$ increases, the Gaussian function decays very fast, which results in a rapid convergence across the virtual cutting curve. In contrast, as $a$ decreases, the Gaussian function spreads out, so that the texture function converges slowly. Since the number of the summation is limited, it is calculated with a threshold. Using this property, this approach provides a continuity control scheme at the boundary. With a low coefficient $a$, the seams on the object are visible while the seams are invisible as $a$ increases. Fig. 5 (a), and (b) show the different results affected by the coefficient values.



(a) coefficient a = 1 (b) coefficient a = 10

Fig. 5. Comparison of a periodized texture function

## 3.3 A Doubly Periodized Texture Function on Genus 1 Objects

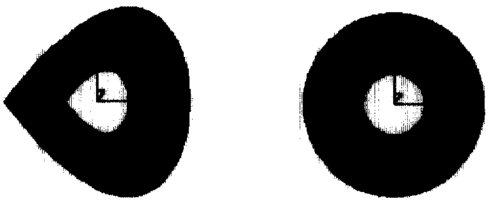
The same approach can be extended to the torus whose domain is 2D. The texture function value [0, c] is set to black(0) and white(1) level. Given a checkerboard texture function on a rectangle plain, the offset from the surface can be mapped to the gray transition from 0 to 1. The process of periodizing function turns out that the resultant has gray level to compromise the difference across the boundary. The texture function on the torus is doubly periodized by using the following formula,

$$O(u,v) = \sum_{k1,k2\in Z}\left[f(u+k1, v+k2)\right] e^{-a(d(u+k1,v+k2))^2} + \sum_{k1,k2\in Z}\left[T(u+k1, v+k2)\right] e^{-a(d(u+k1,v+k2))^2} \frac{n(u,v)}{|n(u,v)|}$$

The normal $n(u, v)$ of the torus at $(u, v)$ can be obtained by calculating the cross product of the directional tangent vectors, that is,

$$n(u,v) = \frac{\partial f(u,v)}{\partial u} \otimes \frac{\partial f(u,v)}{\partial v}$$

Fig. 6 (a) and (b) show the texture mapping, in other words, the scalar field representation using color transition, on the torus.



(a) A texture function (b) A doubly periodized function

Fig. 6. A discontinuous texture function and its periodic function

### 3.4 A Multiperiodized Texture Function on Genus N Objects

Given $T(u, v)=\sin(20u)\sin(20v)+0.5$, most of the approaches for texture mapping on a genus $n$ object is analogous to the texture mapping on torus, except that the texture function $T(u, v)$ is defined on the $4n$-sided polygon in the hyperbolic space. Since the genus $n$ object itself is constructed from the multiperiodic function, it can be represented as

$$F(z) = \sum_{H \in \Gamma_n} f(Hz)\, e^{-a(Hz)^2}, \quad \text{where } z = (u, v),$$

The intensity at the surface point $(x_i, y_i, z_i)$ on the object space is obtained from the texture function $T(z)$ at the corresponded point $z_i = (u_i, v_i)$ in 2D texture domain.

Since the research is dealing with a genus $n$ object, the texture function $T(z)$ is defined on the $4n$ sided polygon in the hyperbolic plane. Fig. 7 shows a texture function on octagon. The resultant texture mapping on the genus $n$ object inevitably generates the visual seems at cross boundary. See Fig. 8. These visible seems due to the discontinuity at the cross boundary would be resolved by incorporating the multiperiodizing function into the texture function. Therefore, the multiperiodized texture function $\overline{T}(z)$ is as follows

$$\overline{T}(z) = \sum_{H \in \Gamma_n} T(Hz)\, e^{-a(Hz)^2}$$

where $\Gamma_n$ is the whole domain consisting of the $4n$-gon. By rescaling the whole range of the texture function onto [0, 1], the offset of the texture function is represented as the intensity of red through blue. An example of a multiperiodized texture function on a deformed double torus is shown in Fig. 9. Notice that the offset of the multiperiodized texture function is gradually being reduced, as it is closer to the cut. This ensures a trade-off relation between the blurred seem and the discontinuity at the cut.

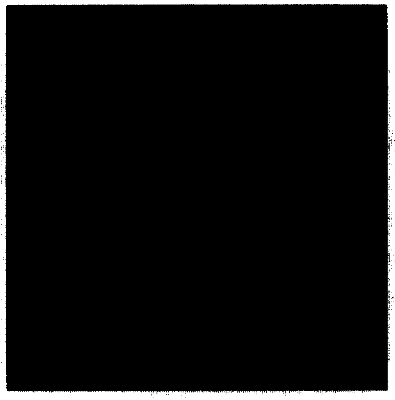


Fig. 7. A texture domain.

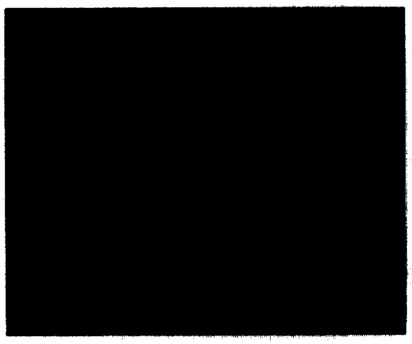


Fig. 8. A textured double torus.

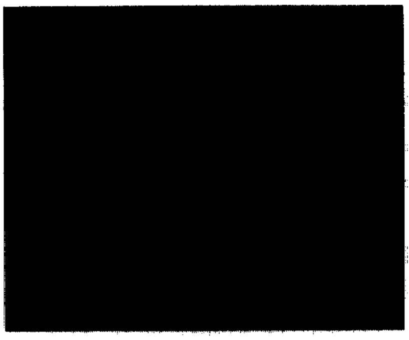


Fig. 9. A seamless textured double torus.

## 4. Bump Mapping Function using Multiperiodic Function on Genus N Objects

Bump mapping which shows the jittered normal vector at each point is one of examples of a vector field visualization. Since a vector in a vector field represents a directional vector, the normal vector at each point can be normalized to a unit vector.

The purpose of this approach is to show that the texture function dealing with vectors on a genus $n$ object can be done by a very simple mapping and the continuity at the boundary can be controlled by users. In order to show the jittered normal on the surface, the scaled sine function is utilized as the texture function. Fig. 10 shows the sine function on the tear-drop curve, which is generated from the equation (2). Since the sine function is not smoothly connected, the normal vector at the junction point is not smooth, either.

$$o(t) = f(t) + T_s(t)\frac{n(t)}{|n(t)|} \tag{2}$$

where $T_s(t) = a\ sin(bt)$ where $a$, $b$ are real

Periodizing the texture function on the tear drop curve, it is

$$O(t) = \sum_{k\in Z}[f(t+k)]\,e^{-a(t+k)^2} + \sum_{k\in Z}[T_s(t+k)]\,e^{-a(t+k)^2}\frac{n(t)}{|n(t)|}$$

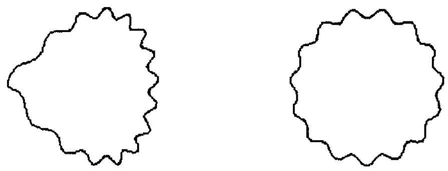
Appropriate normalizing of the function $O(t)$ constructs the infinitely smooth circle with a texture function. Thus, its normal vector is also changed continuously. Fig. 10(a) shows $o(t)$ and Fig. 10 (b), Fig. 11 (a), and (b) shows the periodized bumped circle with several coefficients $a$. Extending this concept to the torus and its homeomorphic surface is straightforward. the following function $O(u, v)$ is obtained.

$$O(u,v) = \sum_{k1,k2\in Z}[f(u+k1, v+k2)]\,e^{-a(d(u+k1,v+k2))^2} +$$

$$\sum_{k1,k2\in Z}[T_s(u+k1, v+k2)]\,e^{-a(d(u+k1,v+k2))^2}\frac{n(u,v)}{|n(u,v)|}$$

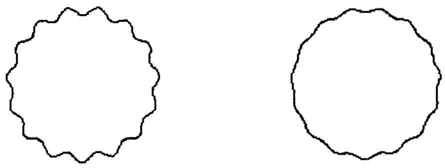
The normal $n(u, v)$ of the torus at $(u, v)$ can be obtained by calculating the cross product of the directional tangent vectors, that is,

$$n(u,v) = \frac{\partial f(u,v)}{\partial u} \otimes \frac{\partial f(u,v)}{\partial v}.$$



(a) a discontinuous texture function (b) periodizing transformation

Fig. 10. Periodizing transformation of a texture function with discontinuous normal vector



(a) coefficient a = 1 (b) coefficient a = 5

Fig. 11. The effects on the coefficient in a Gaussian function

Fig. 12(a) shows the jittered normal vector on the original torus surface before periodizing it. The periodically jittered normal vector on the original torus surface is shown in Fig. 12(b).

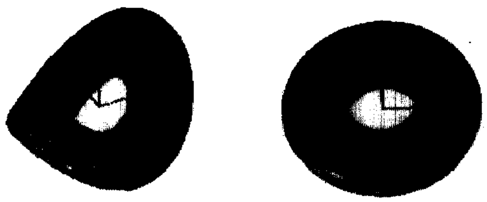


Fig. 12. A bump-mapped torus with a discontinuous texture function and with a periodized texture function

Now the domain is extended into the genus $n$ object. The new surface with a texture function on the multiperiodized genus $n$ object is represented as (3).

$$O(z) = \sum_{H\in\Gamma_n} f(Hz)\,e^{-a(Hz)^2} + T(z)\frac{n(F(z))}{|n(F(z))|} \tag{3}$$

where $\Gamma_n$ is a whole domain consisting of the $4n$-gon. Notice that the bump mapping generated from eq. (3) is not connected smoothly at the cross boundary. This is done by multiperiodizing the texture function and adding it to the object surface in its unit normal direction. Therefore, it is as follows

$$O(z) = \sum_{H\in\Gamma_n} f(Hz)e^{-a(Hz)^2} + \sum_{H\in\Gamma_n} T(Hz)e^{-a(Hz)^2} \frac{n(F(z))}{|n(F(z))|}$$

where $\frac{n(F(z))}{|n(F(z))|}$ is the unit normal vector at the surface point $(x(F(z)), y(F(z)), z(F(z))$ in object space. The normal vector to the perturbed surface is given by taking the cross products of the partial derivatives of $O(z)$:

$$n(O(z)) = n(O(u,v)) = \frac{\partial O}{\partial u} \otimes \frac{\partial O}{\partial v},$$

where

$$\frac{\partial O}{\partial u} = \frac{\partial F(u,v)}{\partial u} + \frac{\partial \overline{T}(u,v)}{\partial u} \frac{n(F(u,v))}{|n(F(u,v))|} + \overline{T}(u,v) \frac{\partial}{\partial u} \frac{n(F(u,v))}{|n(F(u,v))|}$$

$$\frac{\partial O}{\partial v} = \frac{\partial F(u,v)}{\partial v} + \frac{\partial \overline{T}(u,v)}{\partial v} \frac{n(F(u,v))}{|n(F(u,v))|} + \overline{T}(u,v) \frac{\partial}{\partial v} \frac{n(F(u,v))}{|n(F(u,v))|}$$

If $T(u, v)$ is very small compared with the spatial extent of the surface, $\overline{T}(u, v)$ is also very small and the last term in each equation can be ignored. Therefore, it is described as follows

$$n(O(u,v)) = \frac{\partial F(u,v)}{\partial u} \otimes \frac{\partial F(u,v)}{\partial v} + \frac{\partial \overline{T}(u,v)}{\partial u} \left( \frac{n(F(u,v))}{|n(F(u,v))|} \otimes \frac{\partial F(u,v)}{\partial v} \right) + \frac{\partial \overline{T}(u,v)}{\partial v} \left( \frac{n(F(u,v))}{|n(F(u,v))|} \otimes \frac{\partial F(u,v)}{\partial u} \right)$$

The original surface normal $\frac{n(F(u,v))}{|n(F(u,v))|}$ can be obtained from the average of the adjacent triangle normal and its partial derivatives can be obtained from the cross products between the normal vector and an arbitrary unit vector and from the cross products between the normal vector and the new generated vector. These are given in eq. (4) and eq. (5).

$$\frac{\partial F(u,v)}{\partial u} = \frac{n(F(u,v))}{|n(F(u,v))|} \otimes \frac{n(F(u,v)) + 0.1}{|n(F(u,v)) + 0.1|} \quad (4)$$

$$\frac{\partial F(u,v)}{\partial v} = \frac{n(F(u,v))}{|n(F(u,v))|} \otimes \frac{\partial F(u,v)}{\partial u} \quad (5)$$

The partial derivatives of multiperiodic texture function are

$$\frac{\partial \overline{T}(u,v)}{\partial u} = \frac{\partial \sum_{H\in\Gamma_n} T(H(u,v))\, e^{-a(H(u,v))^2}}{\partial u} = \sum_{H\in\Gamma_n} \left( \frac{\partial T(H(u,v))}{\partial u} e^{-a(H(u,v))^2} + T(H(u,v)) \frac{\partial e^{-a(H(u,v))^2}}{\partial u} \right)$$

$$\frac{\partial \overline{T}(u,v)}{\partial v} = \frac{\partial \sum_{H\in\Gamma_n} T(H(u,v))\, e^{-a(H(u,v))^2}}{\partial v} = \sum_{H\in\Gamma_n} \left( \frac{\partial T(H(u,v))}{\partial v} e^{-a(H(u,v))^2} + T(H(u,v)) \frac{\partial e^{-a(H(u,v))^2}}{\partial v} \right)$$

Since the height of texture function $T(H(u, v))$ is small and ignored, then these derivatives are simplified as below:

$$\frac{\partial \overline{T}(u,v)}{\partial u} = \sum_{H\in\Gamma_n} \frac{\partial T(H(u,v))}{\partial u} e^{-a(H(u,v))^2}$$

$$\frac{\partial \overline{T}(u,v)}{\partial v} = \sum_{H\in\Gamma_n} \frac{\partial T(H(u,v))}{\partial v} e^{-a(H(u,v))^2}$$

As a result, the normal vector of a perturbed surface is connected smoothly at cross boundary on the genus $n$ object. Fig. 13 shows the vector field representation of a sine function on the double torus and Fig. 14 and 15 are the multiperiodized sine function on it.

## 5. Conclusion and Future Works

Seamless texture mapping on the smooth genus N object is developed by multiperiodizing a texture function on the 4n-gon and adding it to the object in the direction of the unit normal vector. Existing

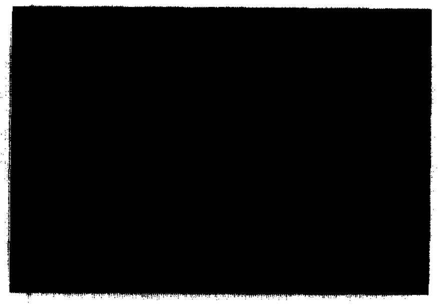


Fig. 13. A bump mapped double torus.

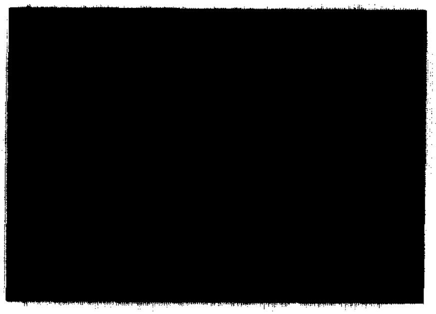


Fig. 14. A seamless bump mapped double torus.

Fig. 15. A seamless bump mapped double torus (homeomorphing object).

texture mapping techniques are based on the patch oriented objects that require very complicated computation for constructing a genus N object. Since the method developed in this paper is based on the global function for a genus N object, it generates texture mapping very fast and easily. In addition, seamless texturing can be manipulated by using multiperiodizing it

Actually, the design approach, the global mapping from the single domain to the genus N object, helps combine the texture mapping on the object. The infinite smoothness of the multiperiodic function eliminates the discontinuity problem at the virtual cut in texture mapping. Periodizing the texture function transforms the discontinuous texture function into a continuous one across the virtual cutting curve, thus, making it seamless.

The applications can be extended into more general areas. For examples, bump mapping is one of applications for normal vector field and checkerboard function is one for scalar field. They are extendable to vector field such as electronic flow and scalar field such as distance and offset. Hence, generalization is one of future works, and environment mapping would also be desirable.

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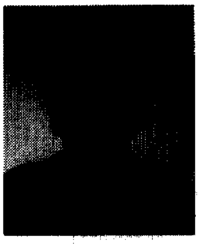

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